

### 3.8.1. Expressive Power Problems

#### A. Languages and Truth Table Coverage.

1. Show that the language  $\{\rightarrow\}$  can build a sentence with the same truth table as “ $(P \vee Q)$ ”.
2. Show that the language  $\{\vee, \leftrightarrow\}$  can build a sentence with the same truth table as “ $(P \wedge Q)$ ”.
3. Show that the language  $\{\rightarrow, \leftrightarrow\}$  can build a sentence with the same truth table as “ $(P \wedge Q)$ ”.

*Hint: use the answers to Problems 1 and 2*

4. Show that the language  $\{\wedge, \rightarrow\}$  can build a sentence with the same truth table as “ $(P \leftrightarrow Q)$ ”.
5. Show that the language  $\{\wedge, \leftrightarrow\}$  can build a sentence with the same truth table as “ $(P \rightarrow Q)$ ”.

**B.** We noted that  $\{\wedge, \vee, \rightarrow, \leftrightarrow\}$  is expressively inadequate. On the basis of that fact – and particularly the evidence which proves that fact – show that if  $\bullet$  and  $\blacktriangle$  are sentences of the Chapter Three language, and  $\blacktriangle$  contains no tildes, then the following are true.

2. If  $\sim\bullet$  is a tautology, then  $\bullet$  contains at least one tilde.
3. If  $\bullet \equiv \sim\blacktriangle$ , then  $\bullet$  contains at least one tilde.<sup>1</sup>

(from Kleene 1967: 20, Problem 4.8, (b) and (c))

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<sup>1</sup> Recall that  $\bullet \equiv \blacktriangle$  means:  $\bullet$  and  $\blacktriangle$  are logically equivalent. See 2.17.